DYNAMICAL THEORY OF SUPERPOSITION OF WAVES

[1]Edison A. Enaibe, (Ph.D.) [2] Daniel A. Babaiwa, (M.Sc.) and [3]John O. A. Idiodi, (Prof.)
1. Department of Physics, Federal University of Petroleum Resources, Effurun, Nigeria.
Phone : +2348068060786, +23408074229081 / E – mail : aroghene 70 @ yahoo.com
2. Department of Science Laboratory Technology, Auchi Polytechnic, Auchi, Nigeria.
Phone: +2348037469046 / E-mail : akinbabaiwa@yahoo.com
3. Department of Physics, University of Benin, Benin City, Edo State, Nigeria.
Phone : +2348037994106 / E – mail : idijohn @ Uniben.edu

Note: All correspondence should be directed to the second author.

Abstract

For the first time, we present the quantitative effects of superposing a 'parasitic wave' on a 'host wave'. The attenuation mechanism of the carrier wave produced by the two interfering waves is effectively studied using simple differential technique. We used the known characteristics of the 'host wave' in the carrier wave to determine the basic characteristics of the interfering 'parasitic wave' which were initially not known. This study reveals that when a carrier wave is undergoing attenuation under any circumstance, it does not consistently come to rest; rather it shows some resistance at some point during the decay process, before it is finally brought to rest. The irregular behaviour exhibited by the carrier wave function during the damping process, is due to the resistance pose by the carrier wave in an attempt to annul the destructive effects of the interfering wave. The spectrum of the characteristic angular velocity and the group angular velocity converge to the same value when the raising multiplier is a maximum and both of them seem to be oppositely related. They both show exemplary behaviour at a certain value of the multiplier. This behaviour is caused by the high attraction or constructive interference of the combined effects of the 'host wave' and the 'parasitic wave'.

Keywords: 'Host wave', 'parasitic wave' carrier wave, characteristic angular velocity, group angular velocity, phase velocity.

1. Introduction

Interference effect that occurs when two or more waves overlap or intersect is a common phenomenon in physical wave mechanics. When waves interfere with each other, the amplitude of the resulting wave depends on the frequencies, relative phases and amplitudes of the interfering waves. The resultant amplitude can have any value between the differences and sum of the individual waves [1]. If the resultant amplitude comes out smaller than the larger of the amplitude of the interfering waves, we say the superposition is destructive; if the resultant amplitude comes out larger than both we say the superposition is constructive.

The interference of one wave say 'parasitic wave' y_1 on another one say 'host wave' y_2 could cause the 'host wave' to decay to zero if they are out of phase. The decay process of y_2 can be gradual, over-damped or critically damped depending on the rate in which the amplitude of the host wave is brought to zero. However, the general understanding is that the combination of y_1 and y_2 would first yield a third stage called the resultant wave say y, before the process of decay sets in. In this work, we refer to the resultant wave as the carrier wave and we think this is a better representation.

A 'parasitic wave' as the name implies, has the ability of destroying and transforming the intrinsic constituents of the 'host wave' to its form after a sufficiently long time. It contains an inbuilt multiplier λ which is capable of raising the intrinsic parameters of the 'parasitic wave' to become equal to those of the 'host wave'. Ultimately, once this equality is achieved, then all the active components of the host wave would have been completely eroded and it ceases to exist.

A carrier wave in this wise, is a corrupt wave function which certainly describes the activity and performance of most physical systems. Thus, the reliability and the life span of most active systems including biological systems are determined by the reluctance and willingness of the active components of the 'host wave' to the destructive influence of the 'parasitic wave'.

Any actively defined physical system carries along with it an inbuilt attenuating factor such that even in the absence of any external influence the system will eventually come to rest after a specified time. This accounts for the non-permanent nature of all physically active matter.

If the wave function of any given active system is known, then its characteristics can be predicted and altered by means of anti-vibratory component. The activity and performance of any active system can be slowed down to zero-point 'dead' by means of three factors: (i) Internal factor (ii) External factor, and (iii) Accidental Factor.

The internal factor is a normal decay process. This factor is caused by ageing and local defects in the constituent mechanism of the matter wave function. This shows that every physically active system must eventually come to rest or cease to exist after some time even in the absence of any external attenuating influence. The internal factor is always a gradual process and hence the attenuating wave function is said to be under-damped.

The external factor is a destructive interference process. This is usually a consequence of the encounter of one existing well behaved active wave function with another. The resultant attenuating wave function under this condition is said to be under-damped, over-damped or critically-damped, depending on how fast the intrinsic constituent characteristics of the wave function decays to zero.

The accidental factor leads to a sudden breakdown and restoration of the active matter wave function to a zero-point. In this case, all the active intrinsic parameters of the matter wave function are instantaneously brought to rest and the attenuation process under this condition is said to be critically-damped.

Generally, we can use the available information of the physical parameters of a wave at any given position and time to determine the nature of its source and the initial characteristics at time $t = 0$, more also, to predict the future behaviour of the wave.

The initial characteristics of a given wave with a definite origin or source can best be determined by the use of a sine wave function. However, for the deductive determination of the initial behaviour of a wave whose origin is not certain, the cosine wave function can best be effectively utilized.

The organization of this paper is as follows. In section 1, we discuss the nature of wave and interference. In section 2, we show the mathematical theory of superposition of two incoherent waves. The results emanating from this study is shown in section 3. The discussion of the results of our study is presented in section 4. Conclusion and suggestions for further work is discussed in section 5. The paper is finally brought to an end by a few lists of references.

1.1 Research methodology

In this work, we superposed a 'parasitic wave' with inbuilt raising multiplier λ on a 'host wave' which also contain an inbuilt lowering multiplier β. The attenuation mechanism of the carrier wave which is the result of the superposition is thus studied by means of simple differentiation technique.

2.0 Mathematical theory of superposition of two incoherent waves

Let us consider two incoherent one-dimensional cosine source functions defined by the non - stationary displacement vectors

$$y_1 = a\,\beta \cos(k\beta\, x - n\beta\, t - \varepsilon\,\beta) \tag{2.1}$$

$$y_2 = b\lambda \cos(k'\lambda x - n'\lambda\, t - \varepsilon'\lambda) \tag{2.2}$$

where all the symbols retain their usual meanings. In this study, (2.1) is regarded as the 'host wave' whose propagation depends on the inbuilt lowering multiplier $\beta\left(=1,\ldots,0\right)$. While (2.2) represents a 'parasitic wave' with an inbuilt raising multiplier $\lambda\left(=0,1,\cdots\right)$. The inbuilt multipliers are both dimensionless and as the name implies, they are capable of gradually lowering and raising the basic intrinsic parameters of both waves respectively with time.

Now, let us superpose (2.2) on (2.1), with the hope to realize a common wave function.

$$y = y_1 + y_2 = a\beta\cos(k\beta\, x - n\beta\, t - \varepsilon\,\beta) + b\lambda\cos(k'\lambda\, x - n'\lambda\, t - \varepsilon'\lambda) \tag{2.3}$$

Suppose, we assume that for a very small parameter ζ, the below equation holds,

$$n'\lambda = \zeta + n\,\beta \tag{2.4}$$

$$y = a\beta\cos(k\,\beta\, x - n\,\beta\, t - \varepsilon\beta) + b\lambda\cos(k'\lambda\, x - n\beta\, t - \zeta\, t - \varepsilon'\lambda) \tag{2.5}$$

Again in (2.5), we let

$$\varepsilon_1' = \zeta\, t + \varepsilon'\lambda \tag{2.6}$$

$$y = a\beta\cos(k\,\beta\, x - n\,\beta\, t - \varepsilon\beta) + b\lambda\cos(k'\lambda\, x - n\beta\, t - \varepsilon_1') \tag{2.7}$$

For the purpose of proper grouping we again make the following assumption:

$$k\,\beta\, x = k'\lambda\, x = \xi \tag{2.8}$$

$$\left(k\beta - k'\lambda\right)x = \xi \tag{2.9}$$

$$y = a\beta\cos\left((\xi - n\,\beta\, t) - \varepsilon\,\beta\right) + b\lambda\cos\left((\xi - n\beta\, t) - \varepsilon_1'\right) \tag{2.10}$$

We can now apply the cosine rule for addition of angles to evaluate each term in (2.10).

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B \tag{2.11}$$

$$y = a\beta\left\{\cos(\xi - n\beta\, t)\cos\beta\,\varepsilon + \sin(\xi - n\beta\, t)\sin\beta\,\varepsilon\right\} +$$

$$b\lambda\left\{\cos(\xi - n\,\beta\, t)\cos\varepsilon_1' + \sin(\xi - n\,\beta\, t)\sin\varepsilon_1'\right\} \tag{2.12}$$

$$y = a\beta\cos(\xi - n\beta\, t)\cos\beta\,\varepsilon + a\beta\sin(\xi - n\beta\, t)\sin\beta\,\varepsilon +$$

$$b\lambda\cos(\xi - n\,\beta\, t)\cos\varepsilon_1' + b\lambda\sin(\xi - n\,\beta\, t)\sin\varepsilon_1' \tag{2.13}$$

$$y = \cos(\xi - n\beta\, t)\left\{a\beta\cos\beta\,\varepsilon + b\lambda\cos\varepsilon_1'\right\} + \sin(\xi - n\beta\, t)\left\{a\beta\sin\beta\varepsilon + b\beta\sin\varepsilon_1'\right\} \tag{2.14}$$

For technicality, let us make the following substitutions

$$A\cos E = a\beta\cos\beta\varepsilon + b\lambda\cos\varepsilon_1' \tag{2.15}$$

$$A\sin E = a\beta\sin\beta\varepsilon + b\lambda\sin\varepsilon_1' \tag{2.16}$$

$$y = A\left\{\cos(\xi - n\beta t)\cos E + \sin(\xi - n\beta t)\sin E\right\} \tag{2.17}$$

$$y = A\cos\left\{\xi - n\beta t - E\right\} \tag{2.18}$$

$$y = A\cos\left\{(k\beta - k'\lambda)x - n\beta t - E\right\} \tag{2.19}$$

The simultaneous nature of (2.15) and (2.16) would enable us to square though them and add the resulting equations term by term, that is

$$A^2\cos^2 E = a^2\beta^2\cos^2\beta\varepsilon + b^2\lambda^2\cos^2\varepsilon_1' + 2ab\beta\lambda\cos\beta\varepsilon\cos\varepsilon_1' \tag{2.20}$$

$$A^2\sin^2 E = a^2\beta^2\sin^2\beta\varepsilon + b^2\lambda^2\sin^2\varepsilon_1' + 2ab\beta\lambda\sin\beta\varepsilon\sin\varepsilon_1' \tag{2.21}$$

$$A^2 = a^2\beta^2 + b^2\lambda^2 + 2ab\beta\lambda\left\{\cos\beta\varepsilon\cos\varepsilon_1' + \sin\beta\varepsilon\sin\varepsilon_1'\right\} \tag{2.22}$$

$$A^2 = a^2\beta^2 + b^2\lambda^2 + 2ab\beta\lambda\cos\left(\beta\varepsilon - \varepsilon_1'\right) \tag{2.23}$$

$$A^2 = a^2\beta^2 + b^2\lambda^2 + 2ab\beta\lambda\cos\left(\beta\varepsilon - (n'\lambda - n\beta)t - \varepsilon'\lambda\right) \tag{2.24}$$

$$A = \sqrt{a^2\beta^2 + b^2\lambda^2 + 2ab\beta\lambda\cos\left((\beta\varepsilon - \varepsilon'\lambda) + (n\beta - n'\lambda)t\right)} \tag{2.25}$$

$$y = \sqrt{a^2\beta^2 + b^2\lambda^2 + 2ab\beta\lambda\cos\left((\beta\varepsilon - \varepsilon'\lambda) + (n\beta - n'\lambda)t\right)} \times \cos\left((k\beta - k'\lambda)x - n\beta t - E\right) \tag{2.26}$$

Upon dividing (2.16) by (2.15), we get that

$$\tan E = \frac{a\beta\sin\beta\varepsilon + b\lambda\sin\varepsilon_1'}{a\beta\cos\beta\varepsilon + b\lambda\cos\varepsilon_1'} \tag{2.27}$$

$$E = \tan^{-1}\left(\frac{a\beta\sin\beta\varepsilon + b\lambda\sin(\varepsilon'\lambda - (n\beta - n'\lambda)t)}{a\beta\cos\beta\varepsilon + b\lambda\cos(\varepsilon'\lambda - (n\beta - n'\lambda)t)}\right) \tag{2.28}$$

Hence (2.26) is the resultant wave function which describes the superposition of the 'parasitic wave' on the 'host wave'. Equation (2.28) is the equation of the non-stationary total phase angle of the carrier wave produced when the 'parasitic wave' interferes with the 'host wave'.

However, without loss of dimensionality, we can recast (2.26) as

$$y = \sqrt{\left(a^2\beta^2 - b^2\lambda^2\right) - 2\left(a\beta - b\lambda\right)^2\cos\left((\beta\varepsilon - \varepsilon'\lambda) + (n\beta - n'\lambda)t\right)} \times \cos\left((k\beta - k'\lambda)x - (n\beta - n'\lambda)t - E\right) \tag{2.29}$$

where

$$A = \sqrt{(a^2\beta^2 - b^2\lambda^2) - 2(a\beta - b\lambda)^2\cos((\varepsilon\beta - \varepsilon'\lambda) + (n\beta - n'\lambda)t)} \tag{2.30}$$

$$E = \tan^{-1}\left(\frac{a\beta\sin\beta\varepsilon + b\lambda\sin(\varepsilon'\lambda - (n\beta - n'\lambda)t)}{a\beta\cos\beta\varepsilon + b\lambda\cos(\varepsilon'\lambda - (n\beta - n'\lambda)t)}\right) \tag{2.31}$$

Equation (2.29) is the generalized carrier wave function which describes the activity and performance of most physically active system. The characteristics intrinsic parameters of the carrier wave function are not stable with time because of the lowering and raising multipliers.

There are three possibilities that would make the carrier wave function given by (2.29) to attenuate to zero and cease to exist:

(i) Natural factor – ageing and local defect which causes under-damping. This situation is applicable in the absence of the 'parasitic wave' $\lambda = 0$ and β is taking a gradual lowering effect.

(ii) Externally induced factor in which case β and λ are both taking oppositely related effects, thereby leading to under-damping, over-damping or critical-damping of the carrier wave function. This situation arises if the intrinsic parameters of the 'parasitic wave' after some time become relatively equal to those of the 'host wave'.

(iii) If by any accident, that is $\lambda = 0$ and suddenly the value of $\beta_{\max} = 0$, then the intrinsic parameters of the 'host wave' goes to zero immediately. This situation is instantaneous and leads to critical damping of the carrier wave function.

Consequently, the existence or the life span of any physically active system which is described by (2.29) is determined by the resistance pose by the intrinsic parameters of the 'host wave' to the destructive influence of the 'parasitic wave' in the carrier wave function.

More also, the response of the parameters of the 'host wave' to its own inbuilt lowering multiplier, even in the absence of the 'parasitic wave', would also determine the liveliness and the life span of the active system it describes.

However, we assume in this study that the intrinsic parameters of the 'host wave' are constant with time, that is, $\beta = 1$, and leave its variation for future research study. Then in consequence, we can rewrite (2.29) – (2.31) as

$$y = \sqrt{\left(a^2 - b^2\lambda^2\right) - 2\left(a - b\lambda\right)^2 \cos\left((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t\right)} \times \cos\left((k - k'\lambda)x - (n - n'\lambda)t - E\right) \tag{2.32}$$

where,

$$A = \sqrt{\left(a^2 - b^2\lambda^2\right) - 2\left(a - b\lambda\right)^2 \cos\left((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t\right)} \tag{2.33}$$

$$E = \tan^{-1}\left(\frac{a\sin\varepsilon + b\lambda\sin(\varepsilon'\lambda - (n - n'\lambda)t)}{a\cos\varepsilon + b\lambda\cos(\varepsilon'\lambda - (n - n'\lambda)t)}\right) \tag{2.34}$$

Thus (2.32) is the required carrier wave function necessary for our study. As the equation stands, it is a corrupt wave function, in which it is only the variation in the intrinsic parameters of the 'parasitic wave' that determines the life span of the physically active system which it describes. This equation describes a propagating carrier wave with non-stationary and frequency dependent amplitude modulated by a spatial oscillating cosine function.

Henceforth, we have agreed in this study, that the initial basic parameters of the 'host wave' are assumed to be constant and also they are initially greater than those of the 'parasitic wave'. Thus by dint of (2.32), the mechanics of a carrier wave can be evaluated with precision.

2.1 The calculus of the total phase angle E of the carrier wave function

Let us now determine the variation of the total phase angle E with respect to time t, since it is a non-stationary function. Thus from (2.34),

$$\frac{dE}{dt} = \left(1 + \left(\frac{a\sin\varepsilon + b\lambda\sin(\varepsilon'\lambda - (n - n'\lambda)t)}{a\cos\varepsilon + b\lambda\cos(\varepsilon'\lambda - (n - n'\lambda)t)}\right)^2\right)^{-1} \times \frac{d}{dt}\left(\frac{a\sin\varepsilon + b\lambda\sin(\varepsilon'\lambda - (n - n'\lambda)t)}{a\cos\varepsilon + b\lambda\cos(\varepsilon'\lambda - (n - n'\lambda)t)}\right) \tag{2.35}$$

$$\frac{dE}{dt} = \left\{\frac{\left(a\cos\varepsilon + b\lambda\cos(\varepsilon'\lambda - (n - n'\lambda)t)\right)^2}{\left(a\cos\varepsilon + b\lambda\cos(\varepsilon'\lambda - (n - n'\lambda)t)\right)^2 + \left(a\sin\varepsilon + b\lambda\sin(\varepsilon'\lambda - (n - n'\lambda)t)\right)^2}\right\} \times$$

$$\frac{d}{dt}\left(\frac{a\sin\varepsilon + b\lambda\sin(\varepsilon'\lambda - (n - n'\lambda)t)}{a\cos\varepsilon + b\lambda\cos(\varepsilon'\lambda - (n - n'\lambda)t)}\right) \tag{2.36}$$

After a lengthy algebra (2.36) simplifies to

$$\frac{dE}{dt} = -Z \tag{2.37}$$

where we have introduced a new variable which is defined by the symbol

$$Z = (n - n'\lambda)\left(\frac{b^2\lambda^2 + ab\lambda\cos((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t)}{a^2 + b^2\lambda^2 + 2ab\lambda\cos((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t)}\right) \tag{2.38}$$

as the characteristic angular velocity of the carrier wave function. It has the dimension of *radian/s.*

We also note that the angular frequency n and n' of both waves are dependent upon the spatial frequency or the wave numbers k and k'. As a result, the total phase angle E would also depend on the spatial frequencies k and k'. Hence, by following the same arithmetic subroutine as in (2.35) - (2.37), we obtain

$$\frac{dE}{d(k - k'\lambda)} = -t\frac{d}{d(k - k'\lambda)}(n - n'\lambda)\left(\frac{b^2\lambda^2 + ab\lambda\cos((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t)}{a^2 + b^2\lambda^2 + 2ab\lambda\cos((\varepsilon - \varepsilon'\lambda) + (n - n'\lambda)t)}\right) \tag{2.39}$$

2.2 Evaluation of the group angular velocity (ω_g) of the carrier wave function

The group velocity is a well-defined but different velocity from that of the individual wave themselves. This is also the velocity at which energy is transferred by the wave [2]. When no energy absorption is present, the velocity of energy transport is equal to the group velocity [3]. The carrier wave function is zero if the average of the spatial oscillatory phase is equal to zero. As a result

$$\cos\left((k - k'\lambda)x - (n - n'\lambda)t - E\right) = 0 \tag{2.40}$$

$$\left((k - k'\lambda)x - (n - n'\lambda)t - E\right) = \frac{\gamma\pi}{2} \quad ; \quad \gamma = 1, 3, 5, \ldots, \tag{2.41}$$

$$x - t\frac{d}{d(k - k'\lambda)}(n - n'\lambda) - \frac{dE}{d(k - k'\lambda)} = 0 \tag{2.42}$$

$$x = t\frac{d}{d(k-k'\lambda)}(n-n'\lambda) + \frac{dE}{d(k-k'\lambda)} \tag{2.43}$$

$$V_g = \frac{x}{t} = \frac{d\omega_g}{d(k-k'\lambda)} \tag{2.44}$$

which is the basic expression for the group velocity, where

$$\omega_g = (n-n'\lambda)\left(\frac{a^2 + ab\lambda\cos\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)}{a^2 + b^2\lambda^2 + 2ab\lambda\cos\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)}\right) \tag{2.45}$$

is the group angular velocity which has the dimension of *radian/s*. Although, ω_g and Z has the same dimension, but where Z depends on time, ω_g is dependent upon the wave number (k).

2.3 Evaluation of the phase or wave velocity (V_p) of the carrier wave function

The phase velocity denotes the velocity of a point of fixed phase angle [3]. At any instant of the wave motion the displacements of other points nearby change also and there will be one of these points, at $x + \delta x$ say, where the displacement $y(x+\delta x, t+\delta t)$ is equal to the original displacement $y(x,t)$ at point x.

These displacements $y(x,t)$ and $y(x+\delta x, t+\delta t)$ will be equal if the corresponding phase angles are equal. The variation of the spatial oscillatory phase of the carrier wave with respect to time gives the phase velocity of the carrier wave. Hence, from (2.40)

$$(k-k'\lambda)\,dx - (n-n'\lambda)\,dt - \frac{dE}{dt}dt = 0 \tag{2.46}$$

$$(k-k'\lambda)\,dx - (n-n'\lambda)\,dt + Zdt = 0 \tag{2.47}$$

$$(k-k'\lambda)\,dx = \left((n-n'\lambda) - Z\right)dt \tag{2.48}$$

$$V_p = \frac{dx}{dt} = \left(\frac{(n-n'\lambda)-Z}{(k-k'\lambda)}\right) \tag{2.49}$$

This has the dimension of *m/s*. Since our argument is equally valid for all values of x, (2.49) tells us that the whole sinusoidal wave profile move to the left or to the right at a speed V_p.

2.4 Calculation of the parameters of the 'parasitic wave' from the known attributes of the 'host wave'.

Let us now consider some arbitrary values of the intrinsic parameters of the 'host wave' contained in the carrier wave function given by (2.32). That is, if we assume say: $a = 0.002\ m$, $n = 5\,rad/s$; $\varepsilon = 0.01746\,rad$ and $k = 1.7456\,rad/m$. Then after a prolonged damping, that is, as the time becomes sufficiently long enough, the intrinsic parameters of both interfering waves become equal to one another and the carrier wave function becomes zero. Based on this simple analysis we get the following relations.

$$a - b\lambda = 0 \quad \Rightarrow \quad a = b\lambda \quad \Rightarrow \quad 0.002 = b\lambda \tag{2.50}$$

$$n - n'\lambda = 0 \quad \Rightarrow \quad n = n'\lambda \quad \Rightarrow \quad 5 = n'\lambda \tag{2.51}$$

$$\varepsilon - \varepsilon'\lambda = 0 \quad \Rightarrow \quad \varepsilon = \varepsilon'\lambda \quad \Rightarrow \quad 0.01746 = \varepsilon'\lambda \tag{2.52}$$

$$k - k'\lambda = 0 \quad \Rightarrow \quad k = k'\lambda \quad \Rightarrow \quad 1.7456 = k'\lambda \tag{2.53}$$

When we divide (2.50) by (2.51); (2.50) by (2.52); (2.51) by (2.52), and finally (2.52) by (2.53), we obtain respectively

$$0.0004\,n' = b\,; \quad 0.1146\,\varepsilon' = b\,; \quad 286.37\,\varepsilon' = n'\,; \quad 0.01k' = \varepsilon' \tag{2.54}$$

With the help of (2.54) we eventually establish by using the rule of simple ratio that the basic intrinsic characteristics of the 'parasitic wave' are :

$$b = 0.00004584 \quad m \quad ; \; n' = 0.1146\,rad\,/\,s \;\; ; \varepsilon' = 0.0004\,rad \;\; \text{and} \;\; k' = 0.04\,rad\,/\,m\,.$$

By using these basic characteristic values in any of (2.50) - (2.53), we generally obtain $\lambda_{\max} = 43.63$. Thus, the physical range of interest of the inbuilt raising multiplier is, $0 \le \lambda \le 44$, where $\lambda(= 0,\, 1,\, 2,\, 3,\, \ldots, 43, \ldots, 43.63)$. We note that at the critical or maximum value of $\lambda_{\max}$, all the intrinsic parameters contained in the carrier wave function would have been correspondingly raised to become equal to one another.

In consequence, we have succeeded in using the available known values of the characteristics of the 'host wave' in the derived carrier wave function to determine the characteristics intrinsic values of the 'parasitic wave' which were initially not known. Also these characteristic values are used to calculate the maximum value of the raising multiplier $\lambda_{\max}$ and hence its subsequent values are determined. The variation in λ is choice dependent but we adopted a slow varying multiplier in such a way that we can understand clearly the physical parameter space which is assessable to the model we have developed.

2.5 Determination of the attenuation constant (η)

Attenuation is a decay process. It brings about a gradual reduction and weakening in the initial strength of the intrinsic parameters of a given active system. In this study, the parameters are the amplitude (a), phase angle (ε), angular frequency (n) and the spatial frequency (k).

The dimension of the attenuation constant (η) is determined by the system under study. However, in this work, attenuation constant is the relative rate of fractional change (FC) in the basic parameters of the carrier wave function.

There are 4 (four) attenuating parameters present in the carrier wave function. Hence,

$$\text{Average. FC,} \;\; \sigma = \frac{1}{4} \times \left(\left(\frac{a - b\lambda}{a} \right) + \left(\frac{\varepsilon - \varepsilon'\lambda}{\varepsilon} \right) + \left(\frac{n - n'\lambda}{n} \right) + \left(\frac{k - k'\lambda}{k} \right) \right) \tag{2.55}$$

$$\eta = \frac{FC\big|_{\lambda = i} - FC\big|_{\lambda = i+1}}{unit\ time\,(s)} = \frac{\sigma_i - \sigma_{i+1}}{unit\ time\,(s)} \tag{2.56}$$

and its dimension is *per second* (s^{-1}). Thus (2.56) gives $\eta = 0.022916\,s^{-1}$ for all values of $i = 1,\, 2,\, 3, \ldots,$.

2.6 Determination of the time (t) (using progressive time)

We used the information provided in section 2.5, to compute the various times taken for the carrier wave to decay to zero. This is possible provided t he value of the time *t* when the raising multiplier is exactly one is known, that is, about the time when λ starts counting.

The maximum time the carrier wave lasted as a function of the raising multiplier λ is also determined with the use of the attenuation equation shown in (2.59). The reader should note that we have adopted a slowly varying regular interval for the raising multiplier $\lambda\ (= 0, 1, 2, 3, \ldots, 43, \ldots, 43.63)$ for our study. The varying interval we adopt will help to delineate clearly the physical parameter space accessible to our model.

However, it is clear from the calculation that the different attenuating fractional changes contained in the carrier wave function are approximately equal to one another.

We can now apply the attenuation equation given below.

$$\sigma = e^{-2\eta t/\lambda} \tag{2.58}$$

$$t = -\frac{\lambda}{2\eta}\ln\sigma \tag{2.59}$$

Clearly, we used (2.59) to calculate the exact value of the time t corresponding to any value of the multiplier λ.

2.7 Evaluation of the velocity of the 'carrier wave function'

Let us now evaluate the velocity with which the entire carrier wave function moves with respect to time. This has to do with the product differentiation of the non-stationary amplitude and the spatial oscillatory cosine phase.

$$\begin{aligned} v = \frac{dy}{dt} &= (a-b\lambda)^2\,(n-n'\lambda)\sin\big((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\big)\times \\ &\Big((a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\big((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\big)\Big)^{-\frac{1}{2}}\times\cos\big((k-k'\lambda)x-(n-n'\lambda)t-E\big)+ \\ &\Big((a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\big((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\big)\Big)^{\frac{1}{2}}\times \\ &\left((n-n'\lambda)+\frac{dE}{dt}\right)\sin\big((k-k'\lambda)x-(n-n'\lambda)t-E\big) \end{aligned} \tag{2.60}$$

It is assumed in this study that after a sufficiently long time and a specific distance covered, the carrier wave ceases to exist, that is, $y \to 0$ as $\lambda \to \lambda_{\max}$. Consequently, the velocity of the carrier wave must also tend to zero at the critical value $\lambda_{\max}$, hence, $v = dy/dt = 0$, and

$$\begin{aligned} &\cos\big((k-k'\lambda)x-(n-n'\lambda)t-E\big)\times \\ &\Big((a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\big((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\big)\Big)^{-\frac{1}{2}} = \\ &-\left((n-n'\lambda)+\frac{dE}{dt}\right)\sin\big((k-k'\lambda)x-(n-n'\lambda)t-E\big)\times \end{aligned}$$

$$\left((a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)\right)^{\frac{1}{2}} \tag{2.61}$$

Further rearrangement of (2.61) with the hope to produce a better result yields

$$(a-b\lambda)^2(n-n'\lambda)\sin\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)\times$$

$$\left(a^2-b^2\lambda^2\right)^{-\frac{1}{2}}\left(1-\frac{2(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\cos\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)\right)^{-\frac{1}{2}}=$$

$$-\left((n-n'\lambda)+\frac{dE}{dt}\right)\tan\left((k-k'\lambda)x-(n-n'\lambda)t-E\right)\times$$

$$(a^2-b^2\lambda^2)^{\frac{1}{2}}\left(1-\frac{2(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\cos\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)\right)^{\frac{1}{2}} \tag{2.62}$$

In qualitative analysis, unlike numerical work, the number one is a fundamental number, an indiscriminate constant value which can only describe the neutral behaviour of a system of varying series. In consequence, the exact behaviour of a non-stationary system may not be studied in the indiscriminate region of a constant value.

Thus the constant value term which is a non-zero-order approximation may therefore be neglected from the varying series solution by direct differentiation of the resulting Binomial equation.

We shall at this stage adopt a new form of approximation technique the "Differentio-Binomial" approximation. This approximation makes use of the second term in the series. The approximation has the advantage of fast convergence and high degree of minimization. The "Differentio-Binomial" approximation is defined as follows.

$$\left(1-x\right)^n=\frac{d}{dx}\left(1-nx-\frac{n(n-1)x^2}{2!}-\frac{n(n-1)(n-2)x^3}{3!}-\cdots\right) \tag{2.63}$$

Thus when we utilize this approximation in (2.66), we get after some simplification

$$-(a-b\lambda)^2(n-n'\lambda)(n-n'\lambda)\sin\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)\times$$

$$\left(a^2-b^2\lambda^2\right)^{-\frac{1}{2}}\left(\frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\right)\sin\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)=$$

$$-\left((n-n'\lambda)+\frac{dE}{dt}\right)\tan\left((k-k'\lambda)x-(n-n'\lambda)t-E\right)\times$$

$$(n-n'\lambda)\left(a^2-b^2\lambda^2\right)^{\frac{1}{2}}\left(\frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\right)\sin\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right) \tag{2.64}$$

$$\frac{(a-b\lambda)^2(n-n'\lambda)}{(a^2-b^2\lambda^2)}\sin\left((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\right)=$$

$$\big((n-n'\lambda)-Z\big)\tan\big((k-k'\lambda)x-(n-n'\lambda)t-E\big) \tag{2.65}$$

$$x=\frac{1}{(k-k'\lambda)}\left\{\tan^{-1}\left(\frac{(a-b\lambda)^2(n-n'\lambda)}{(a^2-b^2\lambda^2)((n-n'\lambda)-Z)}\sin\big((\varepsilon-\varepsilon'\lambda)+(n-n'\lambda)t\big)\right)+(n-n'\lambda)t+E\right\}$$

Thus (2.66) is used to calculate the relative linear distances $x\,(\lambda = 0,\ 1,\ldots,)$, covered by the carrier wave.

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3.0 Results

Table 3.1. Calculated attenuating values of a, ε, n and k of the 'host wave' as they reduce with the growing parameters of the 'parasitic wave' due to the increasing multiplier λ .

S/N i	λ	$(a-b\lambda)$ m	$(n-n'\lambda)$ $rad./s$	$(\varepsilon-\varepsilon'\lambda)$ $rad.$	$(k-k'\lambda)$ $rad./m$	$\left(\frac{a-b\lambda}{a}\right)$	$\left(\frac{n-n'\lambda}{n}\right)$	$\left(\frac{\varepsilon-\varepsilon'\lambda}{\varepsilon}\right)$	$\left(\frac{k-k'\lambda}{k}\right)$
1	0	0.002	5	0.01746	1.7456	1	1	1	1
2	1	0.001954	4.8854	0.01706	1.7056	0.97708	0.97708	0.977090	0.977085
3	2	0.001908	4.7708	0.01666	1.6656	0.95416	0.95416	0.954181	0.954170
4	3	0.001862	4.6562	0.01626	1.6256	0.93124	0.93124	0.931271	0.931256
5	4	0.001817	4.5416	0.01586	1.5856	0.90832	0.90832	0.908362	0.908341
6	5	0.001771	4.4270	0.01546	1.5456	0.8854	0.88540	0.885452	0.885426
7	6	0.001725	4.3124	0.01506	1.5056	0.86248	0.86248	0.862543	0.862511
8	7	0.001679	4.1978	0.01466	1.4656	0.83956	0.83956	0.839633	0.839597
9	8	0.001633	4.0832	0.01426	1.4256	0.81664	0.81664	0.816724	0.816682
10	9	0.001587	3.9686	0.01386	1.3856	0.79372	0.79372	0.793814	0.793767
11	10	0.001542	3.8540	0.01346	1.3456	0.77080	0.77080	0.770905	0.770852
12	11	0.001496	3.7394	0.01306	1.3056	0.74788	0.74788	0.747995	0.747938
13	12	0.001450	3.6248	0.01266	1.2656	0.72496	0.72496	0.725086	0.725023
14	13	0.001404	3.5102	0.01226	1.2256	0.70204	0.70204	0.702176	0.702108
15	14	0.001358	3.3956	0.01186	1.1856	0.67912	0.67912	0.679267	0.679193
16	15	0.001312	3.2810	0.01146	1.1456	0.65620	0.65620	0.656357	0.656279
17	16	0.001267	3.1664	0.01106	1.1056	0.63328	0.63328	0.633448	0.633364
18	17	0.001221	3.0518	0.01066	1.0656	0.61036	0.61036	0.610538	0.610449
19	18	0.001175	2.9372	0.01026	1.0256	0.58744	0.58744	0.587629	0.587534
20	19	0.001129	2.8226	0.00986	0.9856	0.56452	0.56452	0.564719	0.564620
21	20	0.001083	2.7080	0.00946	0.9456	0.54160	0.54160	0.541810	0.541705
22	21	0.001037	2.5934	0.00906	0.9056	0.51868	0.51868	0.518900	0.518790
23	22	0.000992	2.4788	0.00866	0.8656	0.49576	0.49576	0.495991	0.495875
24	23	0.000946	2.3642	0.00826	0.8256	0.47284	0.47284	0.473081	0.472961

Table 3.1 c. t. d. Calculated attenuating values of a , ε , n and k of the 'host wave' as they reduce with the growing parameters of the 'parasitic wave' due to the increasing multiplier λ .

S/N i	λ	$(a-b\lambda)$ m	$(n-n'\lambda)$ $rad./s$	$(\varepsilon-\varepsilon'\lambda)$ $rad.$	$(k-k'\lambda)$ $rad./m$	$\left(\frac{a-b\lambda}{a}\right)$	$\left(\frac{n-n'\lambda}{n}\right)$	$\left(\frac{\varepsilon-\varepsilon'\lambda}{\varepsilon}\right)$	$\left(\frac{k-k'\lambda}{k}\right)$
25	24	0.0009	2.2496	0.00786	0.7856	0.44992	0.44992	0.450172	0.450046
26	25	0.000854	2.135	0.00746	0.7456	0.427	0.427	0.427262	0.427131
27	26	0.000808	2.0204	0.00706	0.7056	0.40408	0.40408	0.404353	0.404216
28	27	0.000762	1.9058	0.00666	0.6656	0.38116	0.38116	0.381443	0.381302
29	28	0.000716	1.7912	0.00626	0.6256	0.35824	0.35824	0.358534	0.358387
30	29	0.000671	1.6766	0.00586	0.5856	0.33532	0.33532	0.335624	0.335472
31	30	0.000625	1.562	0.00546	0.5456	0.3124	0.3124	0.312715	0.312557
32	31	0.000579	1.4474	0.00506	0.5056	0.28948	0.28948	0.289805	0.289643
33	32	0.000533	1.3328	0.00466	0.4656	0.26656	0.26656	0.266896	0.266728
34	33	0.000487	1.2182	0.00426	0.4256	0.24364	0.24364	0.243986	0.243813
35	34	0.000441	1.1036	0.00386	0.3856	0.22072	0.22072	0.221077	0.220898
36	35	0.000396	0.989	0.00346	0.3456	0.1978	0.1978	0.198167	0.197984
37	36	0.00035	0.8744	0.00306	0.3056	0.17488	0.17488	0.175258	0.175069
38	37	0.000304	0.7598	0.00266	0.2656	0.15196	0.15196	0.152348	0.152154
39	38	0.000258	0.6452	0.00226	0.2256	0.12904	0.12904	0.129439	0.129239
40	39	0.000212	0.5306	0.00186	0.1856	0.10612	0.10612	0.106529	0.106324
41	40	0.000166	0.416	0.00146	0.1456	0.0832	0.0832	0.08362	0.08341
42	41	0.000121	0.3014	0.00106	0.1056	0.06028	0.06028	0.06071	0.060495
43	42	7.47E-05	0.1868	0.00066	0.0656	0.03736	0.03736	0.037801	0.03758
44	43	2.89E-05	0.0722	0.00026	0.0256	0.01444	0.01444	0.014891	0.014665
45	43.63	8E-10	2E-06	8E-06	0.0004	4E-07	4E-07	0.000458	0.000229

Table 3.2. Calculated values of the fractional change FC σ, the attenuation constant η, the time t, the total phase angle E, the characteristic angular velocity Z, group spatial angular velocity w_g and the phase velocity V_p of the carrier wave as a function of the multiplier λ and time t.

S/N i	λ	σ	η s^{-1}	t s	E $radian$	Z $radian/s$	w_g $radian/s$	V_p m/s
1	0	1	0	0	0.017460	0	5	2.864345
2	1	0.977084	0.022916	0.50582	0.003270	-0.08956	4.974955	2.916836
3	2	0.954168	0.022916	2.047288	0.034290	-0.21251	4.983314	2.991903
4	3	0.931252	0.022916	4.662173	-0.001960	-0.32888	4.985084	3.066612
5	4	0.908336	0.022916	8.39075	-0.017350	0.355288	4.186312	2.640207
6	5	0.88542	0.022916	13.27605	-0.079010	-0.29453	4.721531	3.054821
7	6	0.862504	0.022916	19.36411	-0.119290	-0.08018	4.392579	2.917494
8	7	0.839588	0.022916	26.70431	0.139488	0.399919	3.797881	2.591349
9	8	0.816671	0.022916	35.34969	0.042058	0.628008	3.455192	2.423675
10	9	0.793755	0.022916	45.35738	0.205710	-0.38454	4.353141	3.141701
11	10	0.770839	0.022916	56.78899	0.191867	0.508648	3.345352	2.486142
12	11	0.747923	0.022916	69.7112	-0.003100	-1.25535	4.994748	3.825634
13	12	0.725007	0.022916	84.19629	0.181847	-1.04681	4.671607	3.691219
14	13	0.702091	0.022916	100.3228	-0.052580	0.789824	2.720376	2.219628
15	14	0.679175	0.022916	118.1766	0.210031	0.705822	2.689778	2.268706
16	15	0.656259	0.022916	137.8512	0.039990	0.838232	2.442768	2.132305
17	16	0.633343	0.022916	159.4495	-0.339340	-0.42882	3.595223	3.25183
18	17	0.610427	0.022916	183.0849	0.143482	0.827066	2.224734	2.087776
19	18	0.587511	0.022916	208.8823	0.430967	-0.33405	3.271248	3.189594
20	19	0.564595	0.022916	236.9806	-0.16544	-1.90056	4.723162	4.792169
21	20	0.541679	0.022916	267.5346	-0.45523	0.153071	2.554929	2.701913
22	21	0.518763	0.022916	300.7176	-0.22809	0.777637	1.815763	2.005038
23	22	0.495847	0.022916	336.7244	0.334186	0.72985	1.74895	2.020506
24	23	0.47293	0.022916	375.7758	-0.48621	-0.75511	3.119315	3.77824

Table 3.2 c. t. d. Calculated values of the fractional change FC σ, the attenuation constant η, the time t, the total phase angle E, the characteristic angular velocity Z, group spatial angular velocity w_g and the phase velocity V_p of the carrier wave as a function of the multiplier λ and time t.

S/N i	λ	σ	η s^{-1}	t s	E $radian$	Z $radian/s$	w_g $radian/s$	V_p m/s
25	24	0.450014	0.022916	418.1231	0.576346	0.331237	1.918363	2.441908
26	25	0.427098	0.022916	464.054	0.615329	0.245121	1.889879	2.534708
27	26	0.404182	0.022916	513.9013	-0.51911	0.530822	1.489578	2.11108
28	27	0.381266	0.022916	568.0518	-0.61295	0.360369	1.545431	2.321862
29	28	0.35835	0.022916	626.9605	0.616602	0.484644	1.306556	2.088484
30	29	0.335434	0.022916	691.167	-0.59416	-1.16001	2.83661	4.843938
31	30	0.312518	0.022916	761.3195	-0.61419	0.474674	1.087326	1.9929
32	31	0.289602	0.022916	838.2066	-0.21821	0.591019	0.856381	1.693792
33	32	0.266686	0.022916	922.8024	0.656477	0.462728	0.870072	1.868711
34	33	0.24377	0.022916	1016.332	-0.11679	0.522556	0.695644	1.634503
35	34	0.220854	0.022916	1120.367	0.592196	0.440871	0.662729	1.718695
36	35	0.197938	0.022916	1236.976	0.797111	0.348156	0.640844	1.854294
37	36	0.175022	0.022916	1368.966	0.373489	-3.4527	4.327096	14.15935
38	37	0.152106	0.022916	1520.285	0.468655	0.339611	0.420189	1.582037
39	38	0.129189	0.022916	1696.763	-0.6595	0.281974	0.363226	1.610045
40	39	0.106273	0.022916	1907.572	-0.24971	0.249177	0.281423	1.516288
41	40	0.083357	0.022916	2168.457	1.176369	0.027355	0.388645	2.669262
42	41	0.060441	0.022916	2510.242	-1.17267	0.08474	0.21666	2.051702
43	42	0.037525	0.022916	3008.273	-1.25995	0.04854	0.13826	2.107623
44	43	0.014609	0.022916	3964.972	1.35794	0.028984	0.043216	1.688106
45	43.63	0.000172	0.022916	8251.37	0.009205	1E-06	1E-06	0.0025

Table 3.3. Calculated values of the distance x, the spatial oscillating phase T, the amplitude A and the displacement y of the carrier wave as functions of the multiplier λ and time t.

$$T = \cos\left((k - k'\lambda)\,x - (n - n'\lambda)\,t - E\right)$$

S/N i	λ	t s	x m	T $radian$	A m	y m
1	0	*0	0.020003	0.999848	0.002000 i	0.00200 i
2	1	0.50582	1.754584	0.868704	0.003172	0.002756
3	2	2.047288	5.705861	0.955981	0.003288	0.003143
4	3	4.662173	13.48270	0.977734	0.003267	0.003194
5	4	8.39075	24.24684	0.937401	0.001431 i	0.001342i
6	5	13.27605	38.31702	0.863438	0.002800	0.002417
7	6	19.36411	55.79767	0.812429	0.002346	0.001906
8	7	26.70431	76.18147	0.832458	0.000878	0.000731
9	8	35.34969	101.1878	0.991752	0.001184 i	0.001174 i
10	9	45.35738	129.7326	0.898923	0.002602	0.002339
11	10	56.78899	162.3820	0.849699	0.001165	0.00099
12	11	69.71120	199.6800	0.999628	0.002866	0.002865
13	12	84.19629	241.1333	0.980405	0.002728	0.002674
14	13	100.3228	287.4581	0.978456	0.000337 i	0.00033 i
15	14	118.1766	338.2618	0.901639	0.001050	0.000947
16	15	137.8512	394.7906	0.998339	0.000309	0.000309
17	16	159.4495	456.6318	0.952214	0.002334	0.002222
18	17	183.0849	524.2336	0.966756	0.000846	0.000817
19	18	208.8823	598.3516	0.958083	0.002230	0.002136
20	19	236.9806	678.5649	0.998341	0.002390	0.002386
21	20	267.5346	766.0553	0.938189	0.001994	0.001871
22	21	300.7176	861.2957	0.943881	0.001239	0.001170
23	22	336.7244	964.2283	0.932100	0.001375	0.001282
24	23	375.7758	1075.660	0.989982	0.002076	0.002055

*The take off time cannot be exactly zero, it could be any value different from zero. This is the correction in time. Hence the model produced a value for the linear distance covered even at t = 0.

Table 3.3. **C. t. d**. Calculated values of the distance x, the spatial oscillating phase T, the amlitude A and the displacement y of the carrier wave as functions of the multiplier λ and time t.

$$T = \cos\left((k - k'\lambda)\, x - (n - n'\lambda)\, t - E\right)$$

S/N i	λ	t s	x m	T $radian$	A m	y m
25	24	418.1231	1197.646	0.950696	0.001803	0.001714
26	25	464.054	1329.26	0.962641	0.001808	0.00174
27	26	513.9013	1471.227	0.945782	0.001605	0.001518
28	27	568.0518	1625.975	0.964079	0.001684	0.001623
29	28	626.9605	1795.618	0.958456	0.001567	0.001502
30	29	691.167	1977.913	0.998764	0.001746	0.001744
31	30	761.3195	2178.933	0.966692	0.001477	0.001428
32	31	838.2066	2399.437	0.988278	0.00119	0.001176
33	32	922.8024	2642.475	0.973368	0.001363	0.001327
34	33	1016.332	2908.959	0.997229	0.001122	0.001119
35	34	1120.367	3207.548	0.980314	0.001218	0.001194
36	35	1236.976	3541.682	0.986987	0.001229	0.001213
37	36	1368.966	3918.18	0.999999	0.001233	0.001233
38	37	1520.285	4350.366	0.99235	0.001011	0.001003
39	38	1696.763	4850.237	0.99264	0.000977	0.000969
40	39	1907.572	5452.397	0.998394	0.000853	0.000852
41	40	2168.457	6203.535	0.999802	0.000829	0.000829
42	41	2510.242	7153.752	0.999756	0.000702	0.000702
43	42	3008.273	8547.186	0.99995	0.000551	0.000551
44	43	3964.972	11235.33	0.99999	0.000341	0.000341
45	43.63	8251.37	64.26845	1.00000	1.79E-06	1.79E-06

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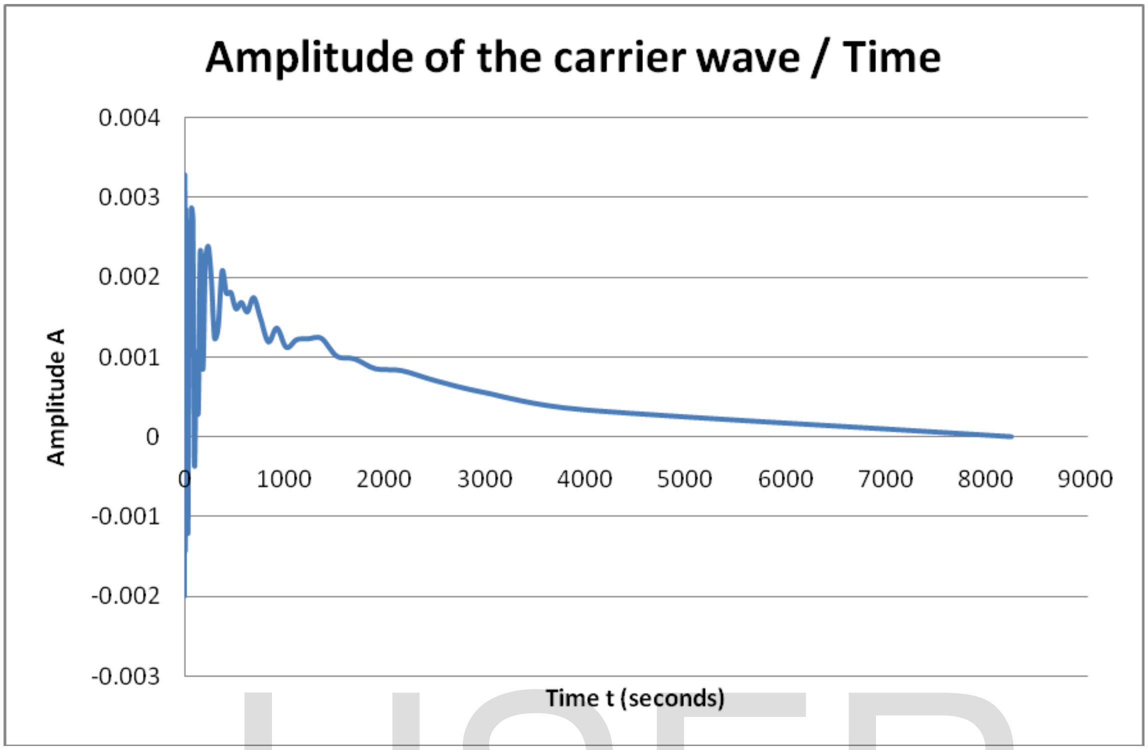

Fig. 1. The value of the amplitude for values of $\lambda = 0, 4, 8, \textit{and } 13$ is assumed to be negative instead of the imaginary value ($i = -1$). Then the values of the amplitude is plotted against time.

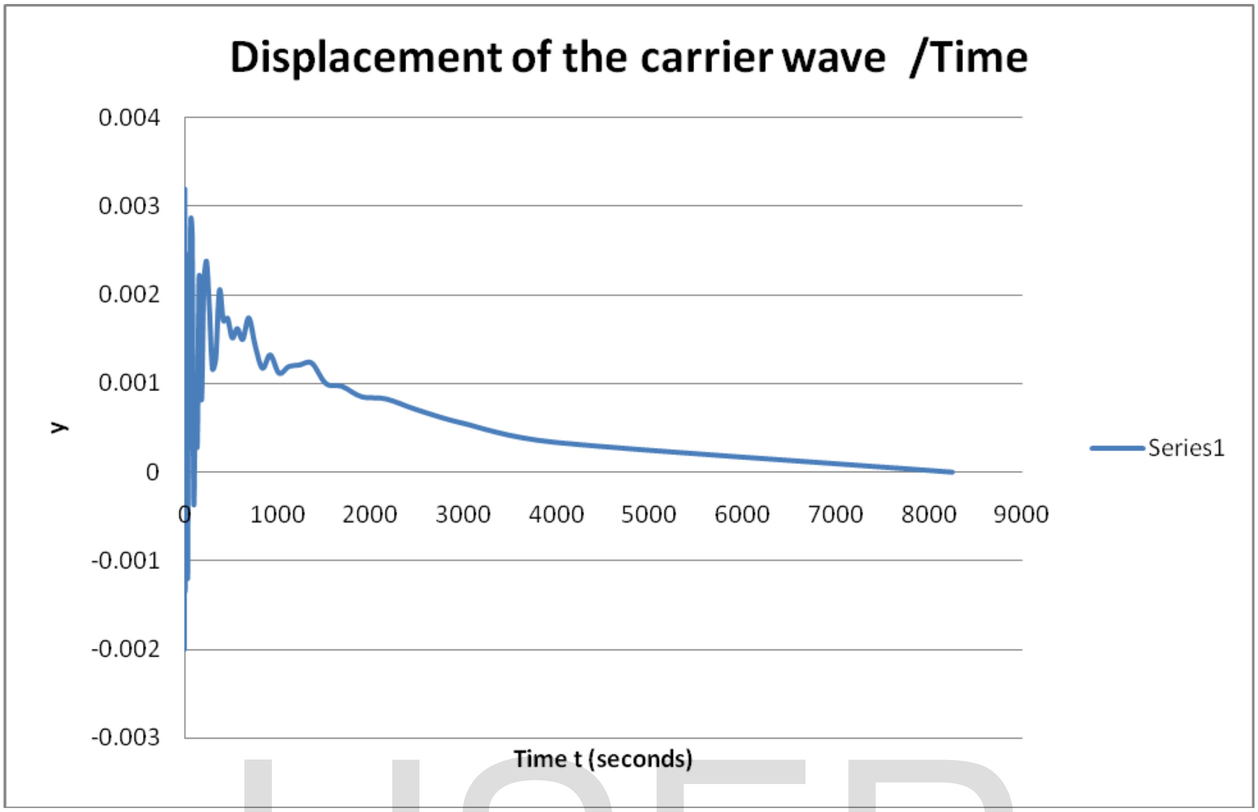

Fig. 2. The value of the displacement vector of the carrier wave y for values of $\lambda = 0, 4, 8, and\ 13$ is assumed to be negative instead of the imaginary value ($i = -1$). Then the displacement is plotted against time.

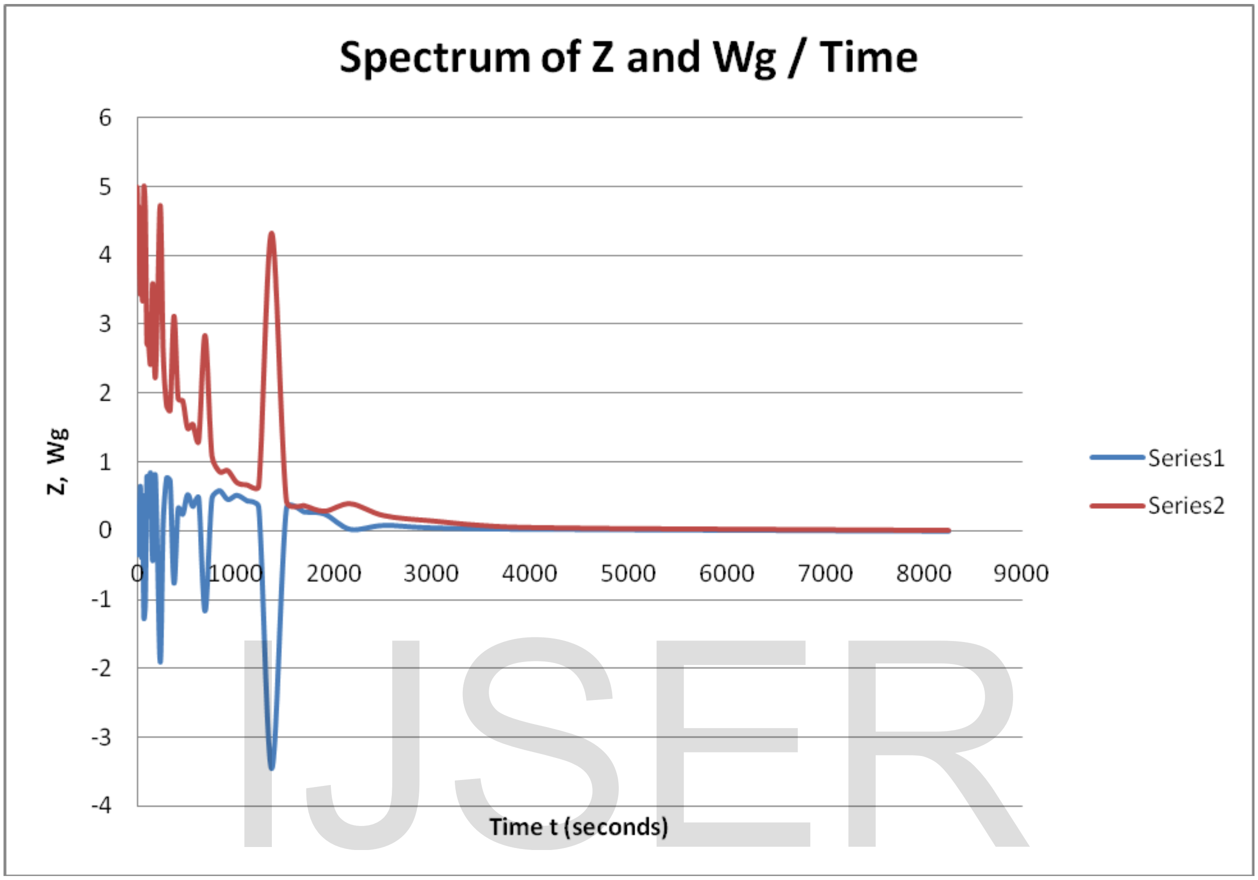

Fig. 3. The spectrum of the characteristic and the group angular velocity of the carrier wave. While the upper series represents the characteristic angular velocity the lower series represents the group angular velocity.

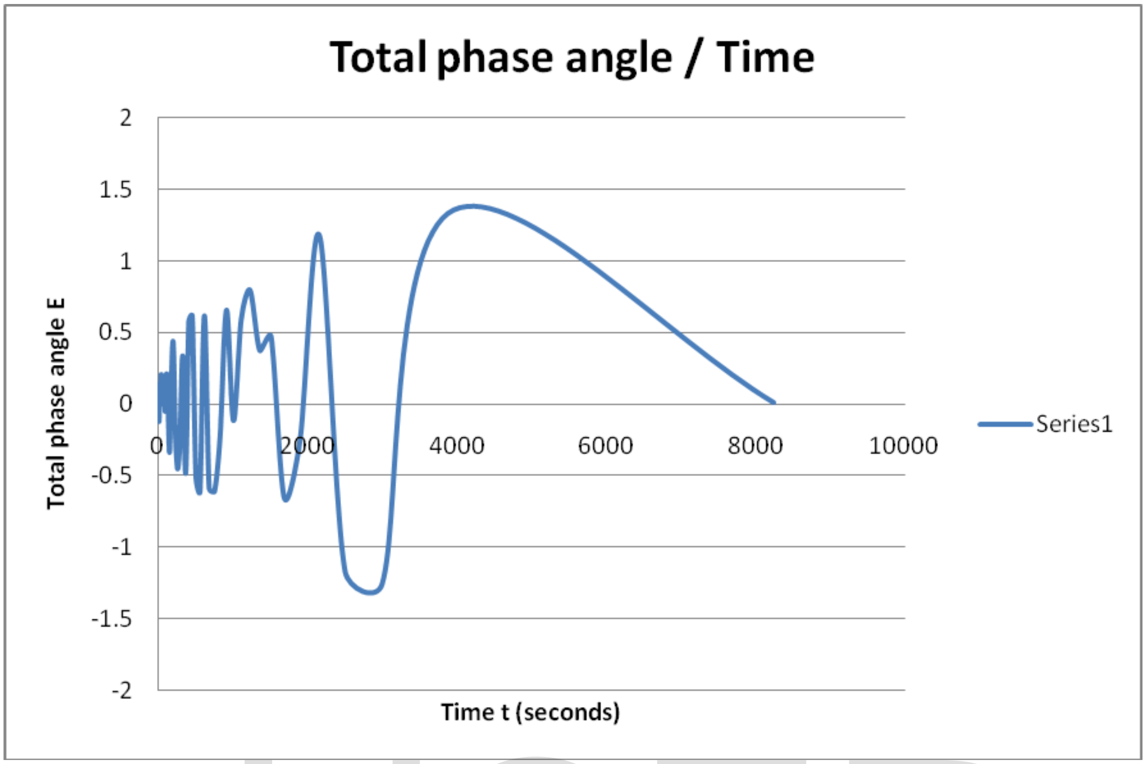

Fig. 4. The spectrum of the total phase angle E of the carrier wave.

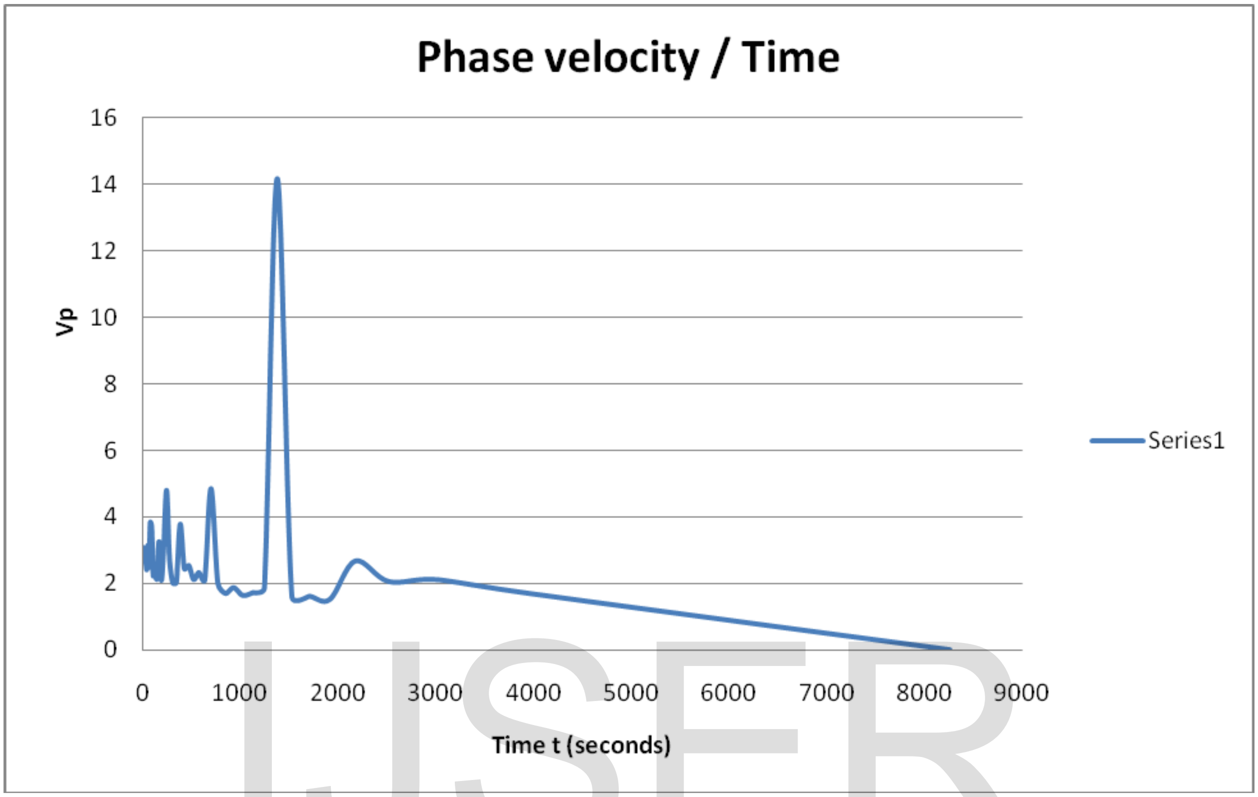

Fig. 5. The spectrum of the phase velocity v_p of the carrier wave.

4.0 Discussion of results

Table 3.1 provides how consistently the constituent parameters of the 'host wave' are correspondingly attenuated to zero by the increasing parameters of the 'parasitic wave'.

The relative fractional change FC of the attenuating parameters as they depend on the raising multiplier is shown in Table 3.2. The FC is approximately equal to one another per given value of the multiplier. The decay process of the total phase angle, the characteristic angular velocity, the group velocity and the phase velocity is not constant. The irregular attenuating behaviour is a consequent of the fact that the amplitude of the carrier wave do not steadily go to zero, rather it fluctuates. The fluctuation is due to the constructive and destructive interference of both the 'host wave' and the 'parasitic wave'. In the regions where the amplitude of the carrier wave is greater than either of the amplitude of the individual wave, we have constructive interference, otherwise, it is destructive.

In Table 3.3, the amplitude is made up of both the imaginary and the real part; $A = A_1 + iA_2$. This shows that the motion is actually two-dimensional (2D). Thus A_1 and A_2 are the components of the amplitude in x and y - directions and A is tangential to the path of the moving amplitude in the carrier wave. The imaginary attenuation in the amplitude of the carrier wave for values of $\lambda = 0,\ 4,\ 8,\ and\ 13$ is unnoticeable or inadequately felt by the physical system described by the carrier wave function. Although, unnoticeable as it may, but so much imaginary destructive harm would have been done to the intrinsic constituent parameters of the 'host wave'.

However, beyond this complex anomalous interval, the amplitude of the carrier wave begins to fluctuate in the interval $14 \le \lambda \le 36$. In this region, the intrinsic parameters of the 'host wave' in the carrier wave function are putting a serious resistance to the destructive influence of the 'parasitic wave'. This resistance is an attempt by the constituent parameters of the 'host wave' to annul the destructive effects of the 'parasitic wave', thereby restoring the system back to the original activity and performance as possessed initially by the 'host wave'.

If the restoring tendency of the constituent parameters of the 'host wave' is not effective enough, then the amplitude of the carrier wave depreciates or decays gradually to zero and it ceases to exist. This occurs in the interval $37 \le \lambda \le 44$.

The trend of event with respect to the displacement of the carrier wave function is similar to that of the amplitude as discussed above. Since the carrier wave function is simply the product of the oscillating amplitude and the spatial oscillating phase. The values of the displacement vector of the carrier wave are less than those of the amplitude. This result is expected since the amplitude is usually the maximum displacement of the wave from the equilibrium position.

It is also shown in Table 3.3, that the respective distance covered by the carrier wave consistently increases in the interval $0 \le \lambda \le 43$, but it decreases drastically when the multiplier reached the critical value $\lambda \le 43.63$. That means the effect of the 'parasitic wave' is now causing a serious retardation on the transport mechanism of the carrier wave after a sufficiently long time. The take off time cannot be exactly zero; it could be any value different from zero. This is the correction in time. Hence the model produced a value for the linear distance covered even at t = 0.

The decay spectrum of the amplitude and the displacement vector of the carrier wave are shown in fig. 1 and 2 respectively. They are both similar and the attenuation process is initially very slow. From the decay spectrum shown in fig. 3, the characteristic angular velocity and the group angular velocity converge to the same value when the multiplier is a maximum and both of them seem to be oppositely related. They show exemplary behaviour for value of λ = 36 at t =1368.996 s.

The spectrum of the decay process of the total phase angle as shown in fig. 4, is maximum at λ = 43 and it begins to decay steadily to zero. Also in fig. 5, the phase velocity behave the same way for the same values of λ = 36 at t =1368.996 s. This is because the characteristic angular velocity is highly attractive in this interval. Thus, the combined effect of the 'host wave' and the 'parasitic wave' are attractive and hence constructive at this value.

5.0 Conclusion

This study shows that the process of attenuation in most physically active system does not obviously begin immediately. The wave function that defines the activity and performance of most active system is guided by some internal factor which enables it to resist any external interfering influence which is destructive in nature. The anomalous behaviour exhibited by the carrier wave function at some point during the damping, is due to the resistance pose by the carrier wave in an attempt to annul the destructive effects of the interfering wave. It is evident from this work that when a carrier wave is undergoing attenuation, it does not steadily or consistently come to rest; rather it shows some resistance at some point in time during the damping process, before the carrier wave function finally comes to rest.

5.1 Suggestions for further work

This study in theory and practice can be extended to investigate wave interference and propagation in two- and three- dimensional systems. The carrier wave function we developed in this work can be utilized in the deductive and predictive study of wave attenuation in exploration geophysics and telecommunication engineering. This work can also be extended to investigate energy attenuation in a HIV/AIDS patient.

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